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QUESTIONS AND DISCUSSIONS.

Edited by W. A. Hurwitz, Cornell University, Ithaca, N. Y.

A number of questions have been standing in this department for some time with no replies or with replies which still leave something to be desired in the way of completeness or finality. Several of these are reprinted below, with the intention of directing our readers' interest to them again, and stimulating replies. To those which have already received some attention short notes are appended, indicating the extent to which they still remain open for consideration.

QUESTIONS.

15. In the *Proceedings of the Royal Society of Edinburgh*, vol. 7, p. 144, in some mathematical notes by Professor P. G. Tait, it is stated:

"If $x^3 + y^3 = z^3$, then $(x^3 + z^3)^3y^3 + (x^3 - y^3)^3z^3 = (z^3 + y^3)^3x^3$.

"This furnishes an easy proof of the impossibility of finding two integers the sum of whose cubes is a cube."

How does this "easy proof" follow?

A partial reply to the above has been received showing that if $x^3 + y^3 = z^3$, then $(x^3 + z^3)^3y^3 + (x^3 - y^3)^3z^3 = (z^3 + y^3)^3x^3$. Can some one show how the "easy proof" then follows?

21. For the Diophantine equation

$$x^2 - y^3 = 17$$

there are known the following solutions:

$$\begin{array}{lrrrrrrrr} x = & 3, & 4, & 5, & 9, & 23, & 282, & 375, & 378661, \\ y = & -2, & -1, & 2, & 4, & 8, & 43, & 52, & 5234. \end{array}$$

One of our readers, who supplied the foregoing facts, desires to know the answers to the following questions: Are there other solutions of the given Diophantine equation? How may all the solutions of this equation be found by a systematic procedure?

In a reply to this question published in the Monthly for June, 1919, E. B. Escott showed how to obtain an infinite number of rational solutions of the equation, and in particular all the integral solutions contained in the above list. References to the literature given in connection with this reply indicate the existence of an infinite number of integral solutions. Thus one question explicitly raised regarding the equation still awaits an answer.

30. A certain Normal University wishes to offer thirty-five hours of college mathematics for the benefit of high-school teachers. What should these courses be in order that, primarily, they may be of the greatest value to high-school teachers of mathematics and, secondarily, that they may furnish stimulus for a more extended pursuit of the subject?

No reply to this question has been received. Some idea of conditions as they are may be obtained from the replies to another question in the Monthly for December, 1916, pp. 395–399.

34. Given the mixed integral and functional equation

$$\int_{x=0}^{x=h} f(x)dx = \frac{h}{6}\left[f(0) + 4f\left(\frac{h}{2}\right) + f(h)\right],$$

to determine the function $f(x)$. This equation is of rather fundamental practical value as it has to do with the most general solid whose volume is given by the prismatoid formula.

In a remark published in the MONTHLY for September, 1917, S. A. Corey showed by reference to a previous paper that the stated relation is satisfied whenever $f(x)$ is a polynomial of degree not exceeding 4. It has not yet been completely proved, however, that this is the only possible form for $f(x)$.

35. Is the following theorem new or has it previously been published?

THEOREM. *If two parallel planes, π and π', cut sections from a cylindrical surface S and two spherical surfaces S_1 and S_2, and if the sum of the sections of S_2 is equal in area to the sum of the sections of S plus the sections of S_1, then the part of S_2 included between π and π' is equal in volume to the sum of the parts of S and S_1 included between π and π'.*

In communicating this question, E. O. Brower commented on the simplicity with which the theorem could be proved, which led him to wonder whether so simple a proposition had remained unnoticed.

37. Criticize the following as fundamental definitions of elementary geometry:

A *plane surface* is the limit approached by a finite portion of the surface of a sphere as the radius increases without limit.

A *straight line* is the limit approached by a finite portion of the circumference of a circle as the radius increases without limit.

38. As the several courses in secondary and collegiate mathematics are now taught there is a noticeable difference of treatment with respect to the relative emphasis placed on logical accuracy and on development of technique. In elementary algebra, technique predominates, while in plane and solid geometry and advanced algebra logic is more strongly stressed. Trigonometry, analytic geometry, and the calculus are less easily classified; but there is at least a general tendency to emphasize logic in analytic geometry and technique in the calculus. It is suggested that a general appraisal of the reasons for this difference in treatment, its value, and possible alterations, would be helpful.

DISCUSSIONS.

It is earnestly desired that the heading "Discussions" in the title of this department shall be something more than a mere name. It is hoped that articles published here will frequently give rise to others on like topics, and that in many cases valuable and stimulating interchanges of views may result. Several papers published as "Discussions" in these columns recently have explicitly requested the opinions of other mathematicians in support or opposition; more than half of the discussions in late numbers are well adapted to bringing out further ideas. Why not read over some of them, write out your opinions, and submit them?

An instance of the sort just referred to is found in the first discussion this month. In November, 1919, Professor Moritz gave a relatively elementary derivation of the formula for the quadrature of the parabola. In the present number, Professor Dunkel presents another way of obtaining the result. Professor Dunkel's work has the merit of extreme simplicity; it also remains in the closest agreement with the definition of "area" and "definite integral" to which we are led in the calculus. On the other hand, the method by which the limit involved in the problem is obtained, while very easy, is distinctly an artifice. A comparison of the two derivations will prove interesting.

In the second discussion Professor Lovitt gives the result of an investigation into the inaccuracies found in the treatment of inverse trigonometric functions in current texts. Every teacher will agree that the statements frequently found may violate all notion of logical precision. An annoying situation of a type less vicious than the cases enumerated by Professor Lovitt is brought about by a text-book which defines the principal value of an inverse function as the smallest positive value.

In the last discussion Professor Rees shows how the motion of a body acted on only by the force of gravity and a resistance proportional to velocity may be readily and easily studied by the use of differential equations and initial conditions in vector form. This paper affords a good instance of the economy, both in notation and actual work, resulting from the use of single vector relations in place of triplets of scalar relations.

I. NOTE ON THE QUADRATURE OF THE PARABOLA.

By OTTO DUNKEL, Washington University.

The article by Professor Moritz entitled "On the Quadrature of the Parabola" in the November issue of the MONTHLY has suggested to the writer to present another derivation of the same result, since, in addition to being fairly simple, this second method follows directly the classic process of defining an area as the common limit of an inferior and a superior sum. This development might be found easy enough to serve as an illustrative example in the presentation of the summation formula in the integral calculus.

Let the equation of the curve be $y = x^m$, $m =$ a positive integer, and suppose that it is desired to obtain the area between the curve, the x-axis and the ordinates at $x = a$ and $x = b$, where b is greater than a and both are positive. Divide the interval from a to b on the x-axis into n subintervals, equal or unequal, and upon them as bases erect two sets of rectangles, the one inscribed and the other circumscribed. The sums of the areas of these rectangles are respectively,

$$I_n = \Sigma x_i^m(x_{i+1} - x_i), \qquad S_n = \Sigma x_{i+1}^m(x_{i+1} - x_i). \tag{1}$$

It will be shown that as n becomes infinite so that the length of the longest subinterval, δ, approaches zero, each of these sums approaches the same limit, which by the usual definition is the area desired. This limit will also be determined in the process.

It may easily be seen from a figure, especially when all the subintervals are equal, that

$$\text{Limit } (S_n - I_n) = 0. \tag{2}$$

This also follows algebraically, for

$$S_n - I_n = \Sigma(x_{i+1}^m - x_i^m)(x_{i+1} - x_i) \leqq \delta\Sigma(x_{i+1}^m - x_i^m).$$

In the latter summation all the terms cancel except the first and last, and hence the difference, $S_n - I_n$, is less than or equal to $\delta(b^m - a^m)$. Thus it follows that (2) is true.